

# Dynamical Stability of Six-Dimensional Warped Brane-Worlds

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## ABSTRACT

We study a generalization of the Randall-Sundrum mechanism for generating the weak/Planck hierarchy, which uses two rather than one warped extra dimension, and which requires no negative tension branes. A 4-brane with one exponentially large compact dimension plays the role of the Planck brane. We investigate the dynamical stability with respect to graviton, graviphoton and radion modes. The radion is shown to have a tachyonic instability for certain models of the 4-brane stress-energy, while it is stable in others, and massless in a special case. If stable, its mass is in the milli-eV range, for parameters of the model which solve the hierarchy problem. The radion is shown to couple to matter with gravitational strength, so that it is potentially detectable by submillimeter-range gravity experiments. The radion mass can be increased using a bulk scalar field in the manner of Goldberger and Wise, but only to order MeV, due to the effect of the large extra dimension. The model predicts a natural scale of  $10^{13}$  GeV on the 4-brane, making it a natural setting for inflation from the ultraviolet brane.

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# 1 Introduction

Brane-world scenarios have undergone considerable theoretical scrutiny in recent times, largely due to the novel solutions which they provide for the gauge-hierarchy problem. Three kinds of such proposals have been made: the ADD scenario [1], in which two extra dimensions are very much larger than ordinary microphysical scales; the intermediate-scale scenario [2], in which the scale of gravity and the compactification scale are very close to one another; and the RS scenario [3], in which the geometry of the extra dimensions is exponentially warped.

The RS scenario differs qualitatively from the other two, with its departure from product-space geometries potentially implying many new kinds of low-energy features, including possible violations of Lorentz-invariance [4], multiple or quasilocalized gravitons [5, 6] and phenomenologically acceptable modifications of the cosmological Friedmann equations [7], among other possibilities.

Most of the effort on exploring this scenario has focused on 5 spacetime dimensions, making it difficult to ascertain which features are generic to warped geometries and which are artifacts of five dimensions. Indeed, some features of these models are very likely to be specific to five dimensions. For instance, the requirement of negative-tension branes – which are generic in five dimensions – does not arise in six dimensions [8, 9]. It is also unlikely that the Friedmann equations are modified in six dimensions in the same way as in five. Moreover, the absence of Kaluza-Klein excitations of the metric’s radion mode is also a 5-dimensional artifact, due to the trivial geometrical nature of 1-D manifolds.

There have been numerous proposals for higher-dimensional generalizations of the RS idea. One of the earliest was to consider intersections of codimension-1 branes as the 3-branes [10]. Others involved modeling the 3-brane where we are supposed to live, or in some cases where gravity is localized, as a cosmic string or higher-dimensional defect [11]. Other relevant work on warped higher dimensional spaces includes [12].

A particularly attractive six-dimensional warped model has been considered in various contexts by several authors [9], [13]–[16]. This model is related to the AdS soliton [17], a double analytic continuation of a planar AdS Black hole metric, and involves two compact dimensions having the topology of a disc with a conical singularity at its centre. The boundary of the disc occurs at a (Planck) 4-brane and a (TeV) 3-brane is placed at the conical defect. The stress energy of the 4-brane requires an anisotropic form which could arise from the smearing of 3-branes around the 4-brane, as suggested in ref. [9], or from Casimir energy of light particles confined to the 4-brane [13].

Since all observable consequences of this (or any other) geometry only involve the theory’s low-energy degrees of freedom, essential to understanding its physical implications is a determination of its low energy spectrum. While this has been partially done in previous references, it is the purpose of this paper to provide a complete accounting of the metric modes, especially as regards the elusive radion mode.

We find results which differ interestingly from what obtains for five-dimensional RS models. Instead of a massless radion, we find that generically the mass squared is nonzero, and possibly negative, depending on details of the 4-brane stress tensor. For a special case involving Casimir

energy on the 4-brane, the radion is an exactly massless modulus at the classical level. Quantum corrections (which we do not here calculate) might stabilize the radion in this case. If the radion mode is stable, the magnitude of the mass squared is of order  $(10^{-3} \text{ eV})^2$ . Both the stable and loop-stabilized cases might therefore make this mode of interest for table-top tests of gravity. The tachyonic instability, if it occurs, does so regardless of the value of the radial size of the extra dimensions in the static solution. We show that it is straightforward to cure this problem in the manner of Goldberger and Wise [18], by adding a bulk scalar field which couples to the branes. The mass of the radion, once stabilized, is suppressed relative to the Planck scale by an additional fractional power of the warp factor, which puts it in the MeV rather than TeV range. Although this could potentially have been problematic, we find that the coupling of the radion is similar to that of gravity, because its wave function is not strongly peaked on the TeV brane. Therefore, although there are cosmological constraints on this model, it is not ruled out by constraints from supernova cooling or radion production in colliders.

We organize our presentation as follows. In section 2 we will introduce the model at the static level. In section 3 we will find the dynamical perturbations for 4-D modes which transform as tensors (gravitons), vectors, and scalars (the radion). We will show that, whereas the tensor and vector modes have a massless ground state, the radion mass squared is generically nonzero and possibly tachyonic, although its magnitude is exponentially small. In section 4 we will show how a bulk scalar field can stabilize the radion mode, and discuss the phenomenology of the model. A summary is given in section 5.

## 2 The AdS Soliton in Randall-Sundrum Models

In this section we present the AdS soliton [17] and its key properties relevant for braneworld applications. A more detailed description of this spacetime including its role in terms of the AdS/CFT correspondence can be found in ref. [17].

The  $(p+2)$ -dimensional AdS Soliton first arose in connection with the AdS/CFT correspondence [19] as a double-analytic continuation of a  $(p+2)$ -dimensional planar AdS black hole metric. In this context the AdS soliton geometry is relevant for the strong coupling description of Lorentzian signature superconformal field theories in which both supersymmetry and conformal invariance have been broken [20]. Specifically for  $p = 3$  and  $p = 5$  it provides a description of strongly coupled  $QCD_3$  and  $QCD_4$  respectively.

### 2.1 The AdS Soliton

In this article we will be interested in the six dimensional (*i.e.*,  $p = 4$ ) AdS soliton for which the line element may be written,

$$ds^2 = a(r) \left( f(r) d\theta^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right) + a^{-1}(r) f^{-1}(r) dr^2 \quad (1)$$

where the metric functions are given by,

$$f(r) = \frac{\rho^2}{L^2} \left( 1 - \frac{\rho^5}{r^5} \right) \quad \text{and} \quad a(r) = \frac{r^2}{\rho^2} \quad (2)$$

and  $\eta_{\mu\nu}$  is the four dimensional Minkowski metric.<sup>1</sup> This is a solution to six dimensional Einstein gravity with negative cosmological constant

$$\Lambda = -\frac{10}{L^2} \equiv -\frac{8}{5}k^2, \quad (3)$$

The spacetime is asymptotically locally AdS as  $r \rightarrow \infty$ ; below we will cut off the radial extent by inserting a 4-brane at a finite value of  $r$ . For convenience we have normalized  $a(\rho) = 1$ , since we will be interested in placing the standard model on a 3-brane situated at that position.

The range of the  $r$  coordinate is  $\rho \leq r < \infty$ , with the geometry smoothly ending at  $r = \rho$  provided that the  $\theta$  coordinate is periodic, with period

$$\beta = \frac{4\pi L^2}{5\rho}. \quad (4)$$

This also requires that, in the context of supergravity, fermions are antiperiodic in the  $\theta$  direction. It is important to note that this geometry is everywhere smooth and nonsingular including at  $r = \rho$  where the circle parameterized by  $\theta$  smoothly shrinks to a point and the geometry ends. An attractive feature of this geometry is that it ends in a natural and nonsingular fashion, allowing constructions similar to those proposed in refs. [21], but which are free of uncontrollable and likely unphysical curvature singularities.

To construct a brane-world model, we will want to imagine that we are living on a (TeV) 3-brane at  $r = \rho$ , thereby introducing a conical defect there of size  $\delta = \kappa^2 \tau_3$ , where  $\tau_3$  is the 3-brane tension and  $\kappa^2$  is related to the six-dimensional Newton's constant by  $\kappa_6^2 = 8\pi G_6$ . This modifies eq. (4) to become

$$\beta = \frac{2L^2}{5\rho} (2\pi - \delta). \quad (5)$$

The extra dimensions are compactified by terminating the space at a 4-brane at  $r = R$ .

In the horospheric coordinate system, the proper distance from  $\rho$  to  $r$  along the  $r$ -direction is given by

$$\tilde{r} \equiv \int_{\rho}^r \frac{dr}{\sqrt{af}} = k^{-1} \cosh^{-1} \left[ (r/\rho)^{5/2} \right], \quad (6)$$

and so  $r/\rho = \cosh(2k\tilde{r}/5) \sim e^{\tilde{r}/L}$  if  $r \gg \rho$ .

We will often find it enlightening to express the solution in polar coordinates,  $\tilde{r}$ , where the line element has the form

$$ds^2 = a(\tilde{r})\eta_{\mu\nu}dx^{\mu}dx^{\nu} + b(\tilde{r})d\tilde{\theta}^2 + d\tilde{r}^2 \quad (7)$$

Here the metric coefficients are given by

$$a(\tilde{r}) = \cosh^{4/5}(k\tilde{r}); \quad b(\tilde{r}) = b_0 \frac{\sinh^2(k\tilde{r})}{\cosh^{6/5}(k\tilde{r})} \quad (8)$$

where  $b_0 = k^{-2}$  if the point  $\tilde{r} = 0$  is regular, and  $\tilde{\theta} \in [0, 2\pi]$ . In general we will suppose the 3-brane has nonvanishing tension located at this point. Then the conical singularity at  $\tilde{r} = 0$

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<sup>1</sup>Our metric's signature is 'mostly plus' and we adopt MTW [22] curvature conventions.

introduces the deficit angle given by  $2\pi(k\sqrt{b_0} - 1)$ . In these coordinates the proper distance between two radii  $\tilde{r}_1$  and  $\tilde{r}_2$  is simply their difference,  $\tilde{r}_2 - \tilde{r}_1$ . We denote the radial position of the 4-brane by  $\tilde{r} = \tilde{R}$ .

We close this section with some comments regarding the stability of the AdS soliton. Since the AdS soliton is constructed from multiple analytic continuations of a black hole space time one might worry about dynamical stability of the solution. In general such analytically continued space times are not always well behaved. For example beginning with the Reissner-Nordstrom black hole in asymptotically flat space one can analytically continue the metric in such a way as to allow for arbitrarily large negative values for the mass parameter. Solutions such as this are inherently unstable *i.e.* small perturbations around the background are tachyonic—see ref. [17] for a detailed discussion.

One of the key results of ref. [17] was that (for  $p = 3$ ) the AdS soliton was found to be perturbatively stable to such linearized fluctuations. Further, in ref. [23] this proof was extended to arbitrary  $p \geq 2$ —see also ref. [24] for a recent discussion. One consequence of this proof is that at least locally within the space of solutions to the Einstein equations with asymptotically locally AdS boundary conditions the AdS soliton represents the minimum energy solution.

It is one of the purposes of this paper to investigate whether the perturbative stability of this space time persists when the geometry is cut off by the introduction of the 4-brane discussed above.

## 2.2 The Gauge Hierarchy

To understand how this model solves the gauge hierarchy problem, let us imagine that all the fundamental scales  $M_6$ ,  $k$ , and  $1/R$  are of order TeV. Then the standard reduction of the gravitational action from 6 to 4 dimensions (using polar coordinates) gives the 4-D Planck mass as

$$\begin{aligned} M_p^2 &= M_6^4 \int d\tilde{r} d\tilde{\theta} a(\tilde{r}) \sqrt{b(\tilde{r})} \\ &\sim \frac{M_6^4}{k^2} a^{3/2}(\tilde{R}) = \frac{M_6^4}{k^2} e^{6k\tilde{R}/5} \end{aligned} \quad (9)$$

Thus by taking  $e^{3k\tilde{R}/5} \sim 10^{16}$ , corresponding to  $k\tilde{R} \cong 60$ , we can explain the largeness of the Planck scale.

Notice that the relation  $M_p^2 \sim a^{3/2}(\tilde{R})(\text{TeV})^2$  differs from the analogous relation in the 5-D RS1 model (*i.e.*, the Randall-Sundrum model which is compactified by the presence of the TeV brane),  $M_p^2 \sim a(\tilde{R})(\text{TeV})^2$ . The additional factor of  $a^{1/2}$  is coming from the  $b^{1/2}$  part of the measure, which gives the size of the extra compact dimension that was not present in the 5-D model. This shows that the present model is a hybrid of the RS and ADD scenarios, in that the hierarchy is due to a combination of warping and having a large extra dimension.

This difference can also be seen by considering the physical mass of a 4-D scalar field which is confined to a 3-brane at a fixed position  $(\tilde{r}, \tilde{\theta})$  in the bulk. Since we are taking the fundamental scale to be TeV, we should assume that its bare mass parameter  $m$  is of this order. But the

physical mass is determined by the usual argument of canonically normalizing its kinetic term:

$$\begin{aligned} S_3 &= -\frac{1}{2} \int d^4x \, a^2 \left[ a^{-1} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \right] \\ &\rightarrow -\frac{1}{2} \int d^4x \left[ \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + a(\tilde{r}) m^2 \phi^2 \right] \end{aligned} \quad (10)$$

Thus the physical mass is given by  $m_3 = m\sqrt{a(\tilde{r})}$ . If we take the particle all the way to the 4-brane, its mass does not reach the Planck scale, but rather a smaller one,  $a(\tilde{R})^{-1/4} M_p \sim 10^{-32/6} M_p \sim 10^{13}$  GeV. This reflects the fact that the strength of gravity is still diluted for a 4-D observer on the 4-brane, by the large extra dimension. We should refer to it as the “ $10^{13}$  GeV brane” rather than the Planck brane.

## 2.3 Properties of the Branes

The 4-brane we need for compactifying the AdS soliton can be constructed by the standard cutting and pasting procedure. Here, the metric in eq. (2) will be cut along the surface  $r = R$  and then pasted onto a mirror image of itself. The resulting space time is then a solution of

$$\kappa^{-2} \sqrt{-G} \left( R_{MN} - \frac{1}{2} R G_{MN} - \Lambda G_{MN} \right) = \sqrt{-g} S_{ab} \delta_M^a \delta_N^b \delta \left( \frac{r-R}{\sqrt{fa}} \right), \quad (11)$$

where  $G_{MN}$  is the metric given in eqn.(2),  $\Lambda = -10L^{-2}$  is the cosmological constant appropriate to six dimensional AdS spaces,  $\kappa^2 = 8\pi G_6$ , and the induced metric on the 4-brane is  $g_{ab} = G_{MN}(R) \delta_a^M \delta_b^N$ .

$S_{ab}$  is the stress tensor of an infinitely thin brane located at the cutting surface. The stress-tensor so defined may be obtained from the Israel matching conditions [25]. A straightforward calculation yields,

$$S_{\mu\nu} = \kappa^{-2} \left( 4 \frac{a'}{a} + \frac{f'}{f} \right) g_{\mu\nu} \quad (12)$$

$$S_{\theta\theta} = \kappa^{-2} \left( 4 \frac{a'}{a} \right) g_{\theta\theta}. \quad (13)$$

Here and in the following uppercase latin indices indicate six dimensional coordinates, while lower case greek indices specify the coordinates parameterizing the directions along the 3-brane. Lower-case latin indices similarly label directions parallel to the 4-brane *i.e.*,  $x^a = (x^\mu, \theta)$ .

A crucial point for this model is that the extra term  $f'/f$  in eq. (12) relative to (13), though small, is nonvanishing, and therefore it is impossible to interpret the 4-brane stress tensor as being due to a pure tension. Were we to do this, thus making  $S_0^0 = S_\theta^\theta$ , the 4-brane would be forced to go to  $r = \infty$ , and we would lose the compactification of the extra dimensions and the localization of gravity.

There are several kinds of physics on the 4-brane which would naturally involve the required difference in  $S_0^0 - S_\theta^\theta$ . One is to imagine that the gluing surface is composed of multiple branes.

As discussed in ref. [9], one could consider the superposition of the stress-energy tensors of a four-brane wrapping the internal circle and a three-brane which is smeared over the internal circle. Indeed eqs. (12,13) then take the form [9],

$$\begin{aligned} S_{\mu\nu} &= \left(T_4 + \frac{T_3}{L_\theta}\right) g_{\mu\nu} \\ S_{\theta\theta} &= T_4 g_{\theta\theta} \end{aligned} \tag{14}$$

where  $L_\theta$  is the proper period of the circle parameterized by  $\theta$  at  $r = R$ .

Another very physical possibility is that the difference between  $T_0$  and  $T_\theta$  is due to the Casimir energy of any massless fields which are confined to the 4-brane [13]. For these the stress-energy tensor will take the form

$$\begin{aligned} S_{\mu\nu} &= \left(T_4 + \frac{c_0}{L_\theta^5}\right) g_{\mu\nu} \\ S_{\theta\theta} &= \left(T_4 - \frac{c_\theta}{L_\theta^5}\right) g_{\theta\theta} \end{aligned} \tag{15}$$

with some dimensionless coefficients  $c_0$  and  $c_\theta$ . To the extent that the trace anomaly vanishes (which is the case at one loop, since the 4-brane is odd-dimensional), the Casimir energy satisfies the condition  $g^{ab}S_{ab} = 0$ , which implies  $c_\theta = 4c_0$ .

In a static background, either of these stress-energy tensors are trivially conserved on the 4-brane. But when we discuss dynamical perturbations of the static space, conservation of stress-energy will yield a nontrivial constraint on the components of  $S_{\mu\nu}$ . This will be discussed in section 3.3, where we show that the ground state of the radion modes is tachyonic for the smeared 3-brane model, but massless for the Casimir model.

The issue of stabilization is closely related to another potential problem with the above models. This concerns the order of magnitude of the difference  $S_0^0 - S_\theta^\theta$ , which is required in order to achieve  $a(R) \sim 10^{21}$  as is needed to solve the hierarchy problem. This requires

$$\frac{S_0^0 - S_\theta^\theta}{S_0^0 + S_\theta^\theta} \sim a(R)^{-5/2} \sim 10^{-53}, \tag{16}$$

which appears to be extremely fine-tuned. From this standpoint, only the Casimir effect can be considered to be natural, since its  $L_\theta^{-5}$  dependence scales precisely like  $a(R)^{-5/2}$ . However the Goldberger-Wise stabilizing field makes it unnecessary to have nonvanishing  $S_0^0 - S_\theta^\theta$ , as was shown by [14]: with the scalar it becomes possible to achieve an exponential hierarchy even when  $S_0^0 = S_\theta^\theta$ . It is interesting that we are able to both determine the size of the extra dimension and stabilize the radion using the same scalar field. In the 5D RS1 model, the two phenomena are necessarily tied together, but not so in 6D. The fact that the size of the extra dimension is determined by the value of  $S_0^0 - S_\theta^\theta$  does not prevent the instability we will demonstrate, so it is not obvious that introducing a new effect to determine the size of  $R$  should stabilize the system. Nevertheless we shall show that it is true.

### 3 Stability Analysis

In the original model of Randall and Sundrum with two branes, fluctuations of the metric were decomposable into Kaluza-Klein modes. Most notably the spectrum included a zero mode which was bound to the brane and served as the graviton in a four dimensional world. The remaining excitations formed a tower of massive modes which were fully five dimensional and had very little support near the brane. In the AdS soliton model presented here we will find a very similar story emerging with a few differences. As in RSI, the spacetime constructed in the previous section is finite and one can view the graviton fluctuations as linearized gravity in a box. This implies that the spectrum of gravitons will again be discrete. Another difference from RSI is that the fluctuations of the metric are now more complicated owing to the greater complexity of the background metric. With only a single extra dimension, the only degree of freedom for the radion mode is the distance between the two branes, since any apparent ripples in the  $dr^2$  metric component can be gauged away by a coordinate transformation. With two or more extra dimensions this is no longer the case, and the radion too has a KK tower of excitations.

By virtue of the symmetries of the geometry at least four of the metric modes must be exactly massless. First, there are two massless states which correspond to the massless 4-D graviton which is ensured by the model's unbroken Lorentz invariance in the directions parallel to the 3-brane. Second there are two states making up the massless 4-D spin-one particle, which is a consequence of the isometry  $\theta \rightarrow \theta + c$  of the extra dimensions.

The counting of massless modes may be further sharpened as follows. If gravity is indeed localized on the 4-brane, we would expect to find a total of five zero modes appropriate to the five independent fluctuations of a massless spin-two particle in  $4 + 1$  dimensions. Since we will find below that the radion generically has a nonvanishing mass in this theory (either tachyonic or real by adding the appropriate scalar), there are in fact only four zero modes bound to the brane. These will have a natural interpretation, at energies below the mass of the radion, as a  $3 + 1$  dimensional graviton and a  $3 + 1$  dimensional massless vector field.

The analysis will proceed by linearizing eq. (11) around the background of the AdS soliton. In particular we will consider fluctuations of the six dimensional metric which are given by  $g_{MN} \rightarrow g_{MN} + h_{MN}$ . A feature here is the fact that  $h_{MN}$  is a tensor and there is thus a variety of polarizations, or graviton modes, that need to be considered. Following refs. [23, 26] we can divide the various polarizations of the six dimensional graviton into three categories. **(i)** Transverse traceless polarizations. These are modes which are polarized in directions parallel to the Lorentz invariant hypersurface spanned by the coordinates  $x^i$ . **(ii)** Vector polarizations. These are gravitons whose polarization is of the form  $\epsilon_{i\theta}$ , *i.e.*, modes polarized along the circle and in the flat piece of the brane. **(iii)** Scalar Polarizations. These are modes which are diagonal but not traceless. It is this last mode which is related to the radion field.

For the cases **(i)** and **(ii)** above we may write the metric fluctuations as  $h_{MN} = H_{MN}(r)e^{ik \cdot x}$  where  $H_{MN}(r)$  is the radial profile tensor and  $k^\mu$  is a 4-dimensional momentum vector with  $k^2 = \eta^{\mu\nu} k_\mu k_\nu = -M^2$ . Further there are ambiguities in the metric perturbations arising from diffeomorphism invariance, which we (partially) handle by imposing a “transverse gauge:”  $H_{M\mu} k^\mu = 0$ . For massive excitations we may always, via the appropriate Lorentz boost, choose



to work in the rest frame<sup>2</sup> so that the momentum can be written as  $k^\mu = \rho \delta_t^\mu$ . In this case, the transversality condition becomes,

$$H_{a\mu}k^\mu = 0 \quad \Rightarrow \quad H_{at} = 0 \quad \forall a \quad (17)$$

Our implicit notation for the 3 + 1-dimensional Minkowski space coordinates is  $x^\mu = (t, x^i)$  with  $i = 1, \dots, 3$ .

We do not consider here the Kaluza-Klein modes<sup>3</sup> corresponding to angular excitations, *i.e.*, around the large extra dimension. One might at first have thought that these had a mass gap of order 10 eV since they are modes which are localized on the 4-brane (assuming they are radially unexcited) and the circumference of the 4-brane is of order  $L_\theta \sim (10 \text{ eV})^{-1}$ . However, if we imagine integrating out only the radial dimension to obtain the effective theory of these modes, we find that the fundamental scale is no longer TeV, but rather  $\sqrt{a(R)} \text{ TeV} \sim 10^{13} \text{ GeV}$ , because of the effect of warping. (The kinetic term of these excitations gets the same exponential factors as does the angular gradient term:  $\mathcal{L} \sim (\dot{\phi})^2/a + (\partial_\theta \phi)^2/b$ .) This effect of warping was alluded to in section 2.2. From this point of view, the size of the compact dimension looks like  $(\text{TeV})^{-1}$ , whose smallness compared to  $\sim 10^{13} \text{ GeV}$  is how the largeness of the extra dimension is manifested. We will leave aside these angular KK modes and instead consider the radial excitations. To determine the spectrum, we must solve eq. (11) with the ansatz (17) as an eigenvalue problem for the mass  $M$ .

The metric fluctuations for case (iii) are considerably more complicated and will be dealt with separately in section 3.3.

### 3.1 Transverse Traceless Modes

As explained above, these modes are polarized parallel to the Lorentz invariant directions on the brane and correspond to

$$H_{\theta M} = H_{rM} = 0 = H_{M\mu}k^\mu \quad \forall M, \quad (18)$$

where the last equality is a restatement of eq. (17).

A consistent solution to eq. (11) linearized around the AdS soliton is provided by the following ansatz,

$$H_{MN} = \varepsilon_{MN} a(r) H(r) \quad (19)$$

where  $\varepsilon_{MN}$  satisfies the conditions in eq. (18) and  $a(r)$  is the metric function appearing in the static solution above.

Solving the equations of motion, which come from linearizing eqn. (11) around our ansatz, imposes that the polarization, must also be traceless,

$$\eta^{\mu\nu} \varepsilon_{\mu\nu} = 0. \quad (20)$$

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<sup>2</sup>Of course when searching for zero modes we are constrained to work with a null momentum vector.

<sup>3</sup>For a discussion of these KK modes as they relate to the stability of the AdS soliton see ref. [23].

Thus eq. (18) describes five independent modes, which can be described as three off-diagonal polarizations, *e.g.*,

$$\varepsilon_{12} = \varepsilon_{21} = 1, \quad \text{otherwise } \varepsilon_{MN} = 0 \quad (21)$$

and two traceless diagonal polarizations, *e.g.*,

$$\varepsilon_{11} = -\varepsilon_{22} = 1, \quad \text{otherwise } \varepsilon_{MN} = 0. \quad (22)$$

For all of these independent polarizations, the radial profile  $H(r)$  satisfies the same differential equation. Substituting the above ansätze, (18) and (19), into eq. (11) and linearizing around the AdS soliton background one finds that the radial profile must satisfy

$$\frac{d^2 H(r)}{dr^2} + \left( 3 \frac{a'(r)}{a(r)} + \frac{f'(r)}{f(r)} \right) \frac{dH(r)}{dr} + \frac{M^2}{f(r)a(r)^2} H(r) = 0 \quad (23)$$

where primes denote differentiation with respect to  $r$ . Here the  $\delta$ -function coming from the right hand side has been canceled exactly by similar terms on the left hand side. One should note that this is *exactly* the equation for the transverse traceless modes originally obtained in ref. [23, 26]—see also ref. [6]. Again, this is independent of the form we choose for the stress tensor, since by exciting these transverse traceless modes we are not perturbing the size of the circle. It is interesting to note that this is precisely the equation describing the propagation of a minimally coupled massless scalar on the AdS soliton background [23, 26].

Here we will determine the eigenvalues numerically using a shooting technique (see ref. [27]). For the purposes of numerical calculation we will henceforth restrict to  $r < R$  and replace the brane by effective boundary conditions on the gravitons at  $r = R$ . Obtaining the correct solution to this problem is equivalent to determining the correct boundary conditions that the radial profile  $H(r)$  must obey at  $r = \rho$  and  $r = R$ . At the brane the absence of  $\delta$ -functions in eq. (23) implies that  $H(r)$  and its first derivative are continuous so requiring an even function of  $r$  gives the boundary condition  $H'(r) = 0$ . The boundary at  $r = \rho$  is more subtle since the metric function  $f(r)$  vanishes there, *i.e.*,  $f(r = \rho) = 0$ . This is reflected in the fact that this point is a regular singular point of eq. (23). So requiring that  $H(r)$  be regular at this boundary gives the condition

$$\left. \frac{dH(r)}{dr} \right|_{r=\rho} = -\frac{L^2 M^2}{5} H(r)|_{r=\rho} \quad (24)$$

It is now straightforward to see that this polarization of the six dimensional graviton indeed has a zero mode. This can be done by setting  $M^2 = 0$  in eq. (23) and integrating directly. After the first integration we find

$$\frac{dH(r)}{dr} = \text{const.} \times a(r)^{-3} f(r)^{-1} \quad (25)$$

which can be seen by inspection to violate the boundary condition at the brane unless the integration constant is forced to vanish. Performing the second integration just leaves the constant solution obeying the above boundary conditions. So we see, referring back to the ansatz in eq. (19), that the physical zero mode for these polarizations is

$$h_{MN} = \varepsilon_{MN} a(r) \quad (26)$$

where  $\varepsilon_{MN}$  obeys the conditions in eq. (19) and we have used our freedom to perform one overall rescaling of the solution to set  $H(r) = 1$ .

In order to obtain the spectrum of nonzero modes we use the numerical shooting technique with the above boundary conditions. The mass eigenvalues are a function of the relative size of the extra dimension  $R/\rho$ , but in the limit that  $R/\rho$  becomes exponentially large, as desired to solve the hierarchy problem, the masses quickly approach their asymptotic values. We give these values for the first few KK modes in the following table. We emphasize that there are no modes with  $M^2 < 0$  and hence no instabilities in this sector.

Mode Number	$M^2 L^2$
0	0
1	16.494
2	44.731
3	85.545
4	138.92
5	204.85

Table 1. Mass squared of the radial graviton KK modes, in units of the AdS curvature radius, in the limit of large warp factor.

Unlike the zero mode, which is localized on the 4-brane, the KK modes are peaked at the TeV brane, a phenomenon which is familiar from the 5D RS model. This behavior is illustrated for the first three modes in figure 1, where the wave functions are plotted.

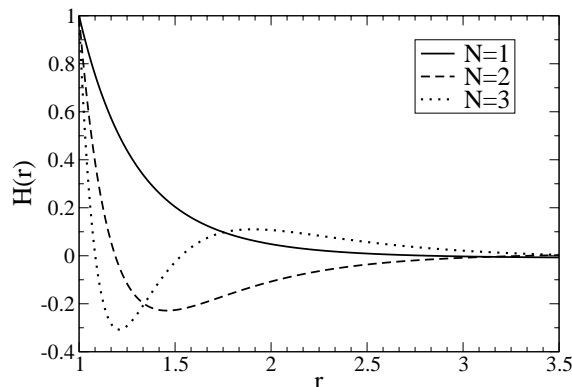


Figure 1. Wave functions for the first three radial KK modes of the graviton.

## 3.2 Vector Modes

The next set of polarizations comes from the same ansatz as in eq. (19); however in this case the polarization tensor is such that it has one leg in the lorentz invariant directions of the four-brane and another on the circle. It has the form

$$\varepsilon_{\theta\mu} = \varepsilon_{\mu\theta} = v_\mu \quad \text{with } k \cdot v = 0 \text{ and } v \cdot v = 1 \quad (27)$$

Polarizations of this form contain three independent modes. Substitution into the equation of motion (11) and linearizing as before yields

$$\frac{d^2 H(r)}{dr^2} + 3 \frac{a'(r)}{a(r)} \frac{dH(r)}{dr} + \frac{M^2}{f(r)a(r)^2} H(r) = -2 \frac{f'(r)}{f(r)} \delta(r-R) H(r) \quad (28)$$

For these modes there is a net contribution from the  $\delta$ -function source term on the right hand side of eq. (11). This can be understood from the fact that the metric perturbation we are considering is a fluctuation in the  $g_{\theta x}$  components of the metric. The corresponding variation of the stress-energy tensor only involves the pieces proportional to  $T_4$  and not those proportional to  $T_3$ . In other words for these modes we have *effectively* that  $\delta T_4 = \delta T_0$  under the fluctuations considered in this section. In our formalism this will manifest itself as a nontrivial boundary condition at  $r = R$ ,

$$\left. \frac{dH(r)}{dr} \right|_{r=R} = \left. \frac{f'(r)}{f(r)} H(r) \right|_{r=R} \quad (29)$$

while enforcing regularity at the singular point  $r = \rho$  requires that  $H(\rho) = 0$ . As with the transverse traceless modes of the previous section, we can obtain an analytic solution for the zero mode of this equation by setting  $M^2 = 0$  and performing the integration directly. We find an exact solution of the form

$$H(r) = -\frac{c}{5} \frac{L^6}{r^5} + b \quad (30)$$

where  $c$  and  $b$  are arbitrary (dimensionful) constants of integration. Imposing the boundary condition in eq. (29) gives

$$b = \frac{c}{5\rho^5} \frac{L^6}{\rho^5} \quad (31)$$

and from this it is straightforward to see that the zero mode is given by

$$H(r) = \frac{cL^8}{5\rho^7} f(r) \quad (32)$$

Fortuitously this solution also satisfies the boundary condition at  $r = \rho$  for all values of  $c$  since this is precisely where  $f(r) = 0$ . Choosing to normalize the wavefunction so that  $H(R) = 1$  amounts to choosing  $c = 5 \frac{\rho^7}{L^8} f(R)^{-1}$ . Returning to the ansatz in eq. (27) we see that the physical zero mode takes the simple form

$$H_{\mu\tau} = \varepsilon_{\mu\tau} a(r) \frac{f(r)}{f(R)} \quad (33)$$

which is indeed peaked at  $r = R$ . In order to analyze the nonzero modes we again turn to numerics and find a positive definite spectrum which implies that no instabilities are caused by exciting these vector modes. In the table below we again present, in the limit of large warp factor, the first few eigenvalues in this spectrum.

Like the spin-2 modes, the vector KK modes are also peaked near the TeV brane, although their wave function vanishes precisely there. The first three modes are shown in figure 2.

Mode Number	$M^2 L^2$
0	0
1	25.001
2	59.752
3	106.91
4	166.61
5	238.80

Table 2. Mass squared of the radial graviphoton KK modes, in units of the AdS curvature radius, in the limit of large warp factor.

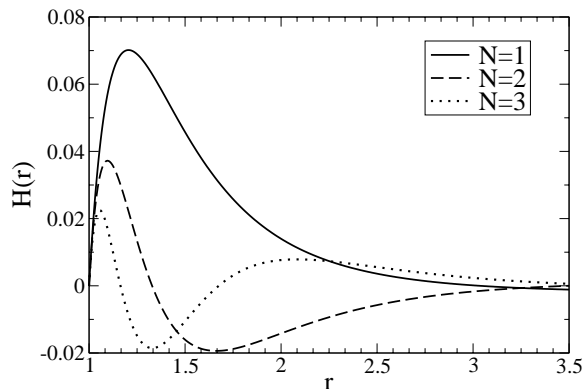


Figure 2. Wave functions for the first three radial KK modes of the graviphoton.

### 3.3 Scalar Modes

Here we will consider modes which fall into category **(iii)** above. There is in fact only a single mode (at the lowest KK level), although it corresponds to simultaneous fluctuations of different components of the metric. Most importantly this mode includes fluctuations of both  $g_{\theta\theta}$  and  $g_{rr}$  and thus couples fluctuations of the radial size to fluctuations in the size of the compact circle. For the radion, it is convenient to switch to the polar coordinates introduced in eq. (7), where the equations take a somewhat simpler form. We start by writing the perturbed metric as

$$ds^2 = a(\tilde{r}, t) \eta_{\mu\nu} dx^\mu dx^\nu + b(\tilde{r}, t) d\tilde{\theta}^2 + c(\tilde{r}, t) d\tilde{r}^2 \quad (34)$$

where the perturbation around the static background corresponds to

$$\begin{aligned} a(\tilde{r}, t) &= e^{-A_0(r) - A_1(r, t)} \equiv a_0(\tilde{r}) e^{-A_1(\tilde{r}, t)} \\ b(\tilde{r}, t) &= e^{-B_0(\tilde{r}) - B_1(\tilde{r}, t)} \equiv b_0(\tilde{r}) e^{-B_1(\tilde{r}, t)} \\ c(\tilde{r}, t) &= 1 + C_1(\tilde{r}, t) \end{aligned} \quad (35)$$

and we have now included a subscript ‘0’ to denote the metric functions of the background around which we are expanding, *i.e.*, the AdS soliton. Here we have gone to the 4-D rest frame

of the fluctuation

$$A_1(\tilde{r}, t) = A_1(\tilde{r}) \text{Re}(e^{-i\omega_r t}) \quad (36)$$

and similarly for  $F_1$  and  $G_1$ . This is generically valid since for arbitrary values of the parameters there will be no zero mode. We will find however that there is a special case in which there is a zero mode and hence no rest frame. For this case we have checked that the procedure outlined here gives the correct result.

We can unify the models of the 4-brane stress energy at  $\tilde{r} = \tilde{R}$  which were discussed in section 2.3 by writing

$$\begin{aligned} S_{\mu\nu} &= \left( T_4 + \frac{T_3}{L_\theta^\alpha} \right) g_{\mu\nu} \equiv V_0 g_{\mu\nu} \\ S_{\theta\theta} &= \left( T_4 - \frac{T'_3}{L_\theta^\alpha} \right) g_{\theta\theta} \equiv V_\theta g_{\theta\theta} \end{aligned} \quad (37)$$

where  $L_\theta = \int d\theta \sqrt{b}|_{\tilde{r}=\tilde{R}}$  is the circumference of the compact 4-brane dimension. The smeared 3-brane model has  $\alpha = 1$  and  $T'_3 = 0$ , while the Casimir effect model has  $\alpha = 5$  and  $T_3, T'_3 \neq 0$ . Notice that the metric tensor elements appear within the definition of the circumference  $L_\theta$ . Also  $T_3$  really has the dimensions of a 3-brane tension only when  $\alpha = 1$ .

We can now write the Einstein equations for this metric ansatz. Since it will later be necessary to add a bulk minimally-coupled scalar field, we include it here, although at first we shall carry out the analysis with no scalar. Ignoring terms like  $\dot{a}^2$  which would be higher order in the perturbations, the  $(tt)$ ,  $(tt) + (ii)$ ,  $(rr)$ ,  $(\theta\theta)$  and  $(tr)$  components of the Einstein equations are

$$\begin{aligned} \frac{3}{2} \frac{a''}{a} + \frac{3}{4} \frac{a' b'}{a b} - \frac{3}{4} \frac{a' c'}{a c} + \frac{1}{2} \frac{b''}{b} - \frac{1}{4} \left( \frac{b'}{b} \right)^2 - \frac{1}{4} \frac{b' c'}{b c} + \\ = -\kappa^2 \left( c[\Lambda + V(\phi)] + \frac{1}{2} \phi'^2 + V_0 \sqrt{c} \delta(\tilde{r} - \tilde{R}) + T_b \sqrt{\frac{c}{b}} \delta(\tilde{r}) \right) \end{aligned} \quad (38)$$

$$2 \frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} = 0 \quad (39)$$

$$\frac{3}{2} \left( \frac{a'}{a} \right)^2 + \frac{a' b'}{a b} - \frac{1}{2a_0} \left( \frac{\ddot{b}}{b} + 3 \frac{\ddot{a}}{a} \right) = -\kappa^2 \left( c[\Lambda + V(\phi)] - \frac{1}{2} \phi'^2 \right) \quad (40)$$

$$2 \frac{a''}{a} + \frac{1}{2} \left( \frac{a'}{a} \right)^2 - \frac{a' c'}{a c} - \frac{1}{2a_0} \left( 3 \frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} \right) = -\kappa^2 \left( c[\Lambda + V(\phi)] + \frac{1}{2} \phi'^2 + V_\theta \sqrt{c} \delta(\tilde{r} - \tilde{R}) \right) \quad (41)$$

$$6 \frac{\dot{a}'}{a} - 6 \frac{\dot{a} a'}{a a} - \frac{\dot{b} a'}{b a} - 3 \frac{\dot{c} a'}{c a} + 2 \frac{\dot{b}'}{b} - \frac{\dot{b} b'}{b b} - \frac{\dot{c} b'}{c b} = 4\kappa^2 \dot{\phi} \phi' \quad (42)$$

where  $T_b$  is the tension of the 3-brane at  $\tilde{r} = 0$  and primes denote  $\partial_{\tilde{r}}$ .

The next step is to linearize the field equations in the dynamical perturbations. We use the relation  $\ddot{A}_1 = -m_r^2 A_1$ , and similarly for the other perturbations, where  $m_r$  is the sought-for radion mass. Expanding the  $(tt) + (ii)$ ,  $(tr)$  and  $(rr)$  Einstein equations, respectively, to first order gives

$$m_r^2 (2A_1 + B_1 - C_1) = 0 \quad (43)$$

$$4(A'_1 - \frac{1}{2}A'_0C_1) + \left[ B'_1 - A'_1 - \frac{1}{2}(B'_0 - A'_0)(B_1 + C_1) \right] - 2\kappa^2\phi'_0\phi_1 = 0 \quad (44)$$

$$(3A'_0 + B'_0)A'_1 + A'_0B'_1 - A'_0 \left( \frac{3}{2}A'_0 + B'_0 \right) C_1 - \frac{m_r^2}{2a_0}(3A_1 + B_1) + \frac{\kappa^2}{2} \left[ \phi_0'^2 C_1 + 2 \frac{\partial V}{\partial \phi} \phi_1 - 2\phi'_0\phi'_1 \right] = 0 \quad (45)$$

and similarly for the scalar field we find

$$\phi_1'' - (2A'_0 + B'_0)\phi_1' - \frac{1}{2}(4A'_1 + B'_1 + C'_1)\phi_0' - \left( C_1 \frac{\partial V}{\partial \phi} + \frac{\partial^2 V}{\partial \phi^2} \phi_1 - \frac{m_r^2}{a_0} \phi_1 \right) = 0. \quad (46)$$

We note that even for a massless mode, the energy is nonvanishing, so that eq. (43) would still provide a constraint among the components of the perturbation (we would then have to consider its spatial momentum too). Moreover (44) is the time-integrated form of the  $(tr)$  equation, which we have organized in a form whose purpose will become clear momentarily. We have not written the  $(tt)$  nor  $(\theta\theta)$  components since these are not independent equations:  $(tt)$  can be obtained from  $(tr)'$  and  $(tt) + (ii)$ , which follows from the Bianchi identities; and  $(\theta\theta)$  follows from combining  $(tr)$ ,  $(rr)$ ,  $(rr)'$  and  $\phi$  equations in a manner which is not obvious, but which could be anticipated, since first order constraint equations, like  $(rr)$  and  $(tr)$ , must be consistent with the second-order dynamical equations,  $(tt)$ ,  $(ii)$  and  $(\theta\theta)$ .

For the remainder of this section, we will assume there is no bulk scalar field present. Effects of the bulk scalar will be considered in the next section.

The boundary conditions at the 4-brane come from integrating over the delta functions to find the discontinuity in the first derivatives, and using  $Z_2$  symmetry across the brane to identify, *e.g.*,  $\Delta a'/a = -2a'/a$  at  $\tilde{r} = \tilde{R}$ . At zeroth order in the perturbations, this gives the jump conditions (12,13) for the static solution. Expanding to first order, and assuming that  $T_3$  is a constant (more about this below) we find

$$A'_1 = \frac{1}{2}A'_0C_1 - \frac{\alpha}{8} \left( \frac{T'_3}{T_3 + T'_3} \right) (B'_0 - A'_0)B_1 \quad (47)$$

$$B'_1 - A'_1 = \frac{1}{2}(B'_0 - A'_0)(C_1 + \alpha B_1). \quad (48)$$

In addition, we must consider the boundary condition at the 3-brane at the center,  $\tilde{r} = 0$ . With two extra dimensions, the effect of such a point-like source is to introduce a conical singularity and a corresponding deficit angle, as we have discussed in section 2.1. Since the defect is unchanged by perturbations around the static solution, we insist that the deficit angle does not vary. If we consider a circle with  $\tilde{r} = \epsilon$  around the 3-brane, with circumference  $L$  and physical radius  $D$ , we therefore demand that  $L/D$  be invariant in the limit that  $\epsilon \rightarrow 0$ :

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \delta \left( \frac{L}{D} \right) &= \lim_{\epsilon \rightarrow 0} \delta \left( \frac{\int d\tilde{\theta} \sqrt{b}}{\int_0^\epsilon \sqrt{c}} \right) \\ &= -\frac{L}{2D} (B_1 + C_1)|_{\tilde{r}=0}, \end{aligned} \quad (49)$$

in other words,  $B_1 + C_1 = 0$  at  $\tilde{r} = 0$ . In addition, we expect  $A'_1$  to vanish at  $\tilde{r} = 0$ . The latter can be shown to be satisfied from  $B_1 + C_1 = 0$  combined with the bulk equations of motion, so it gives no additional constraint.

We must pause to discuss an important point, concerning the counting of boundary conditions versus independent differential equations. Using the algebraic constraint (43) to eliminate  $C_1$ , we have two first order o.d.e.'s for  $A_1$  and  $B_1$ . On the face of it, our system looks overconstrained: there are three boundary conditions! But in fact the system is not overconstrained. Rather, there is a constraint on the stress-energy tensor on the 4-brane, due to its conservation. By computing  $\sum_{a \neq \tilde{r}} S^{0a}_{;a} \equiv 0$ , where  $S_{ab}$  is the surface stress energy on the 4-brane, we obtain

$$\frac{dT_3}{dt} = \frac{\dot{B}_1}{2} (T_3(1 - \alpha) + T'_3) \quad (50)$$

Integrating this, we see that unless the constraint

$$T'_3 = (\alpha - 1)T_3 \quad (51)$$

is satisfied, then  $T_3$  must have had extra hidden dependence on  $B$  (hence the circumference of the circle  $L_\theta$ ) beyond that which was explicitly assumed. If  $T_3$  is truly constant, then any physical model of the stress-energy must satisfy (51). This is true for the model which corresponds to delocalizing (smearing) a 3-brane around the circular dimension of the 4-brane, since there  $\alpha = 1$  and  $T'_3 = 0$ . And it tells us that the Casimir energy model with  $\alpha = 5$  must satisfy  $T'_3 = 4T_3$ , as we saw earlier was indeed the case for massless particles. With any such choice, it is easy to see that there are not really two boundary conditions at the 4-brane; rather, imposing the first b.c. (47), together with the  $(tr)$  equation (44), implies the second b.c., (48). The result of this discussion is that it suffices to impose just one b.c. at  $\tilde{r} = \tilde{R}$ , say (47), which using (51) can be written more simply as

$$A'_1 = \frac{1}{2}A'_0C_1 - \frac{(\alpha - 1)}{8}(B'_0 - A'_0)B_1. \quad (52)$$

We solved the above system of equations for the radion numerically, for the case of  $\alpha = 1$  (the general result for arbitrary values of  $\alpha$  will be given below) and we find that it has a negative value of  $m_r^2$ —it is a tachyon. The graph of its dependence on the ratio of warp factors between the 4-brane and the 3-brane is shown in figure 3a. If we normalize  $a(0) = 1$  at the 3-brane, then in the regime where the hierarchy is large (right hand side of the graph), the radion mass squared depends on  $a(\tilde{R})$  ( $a$  evaluated at the 4-brane) as

$$m_r^2 \cong -20 L^{-2} a(\tilde{R})^{-3/2} \quad (\alpha = 1 \text{ case only}), \quad (53)$$

where we recall that  $L$  is the AdS curvature radius. Since  $L \sim 1/\text{TeV}$  and  $a(\tilde{R})^{3/2} \cong 10^{32}$  to solve the hierarchy problem, we obtain

$$\sqrt{-m_r^2} \sim 10^{-3} \text{eV}. \quad (54)$$

This is well above the present Hubble scale, so we would have noticed the expansion or contraction of the extra dimension due to the change in the strength of gravity, if eq. (54) were true.



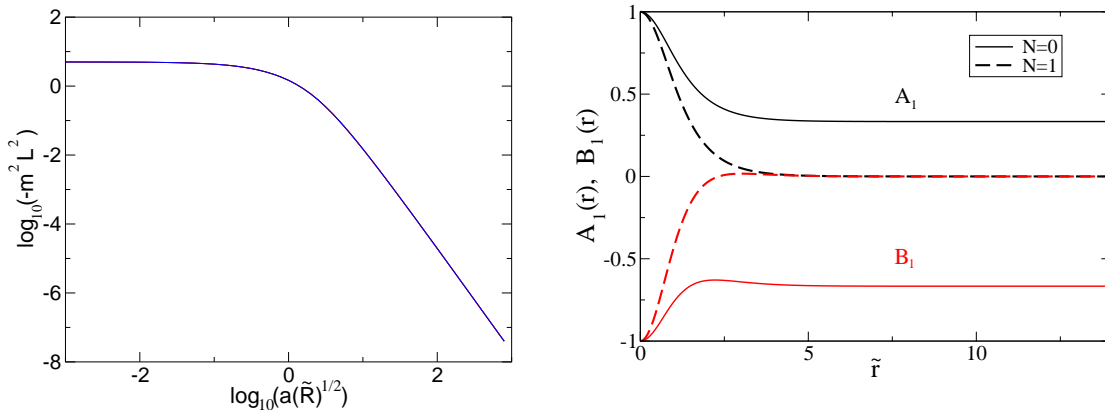


Figure 3. (a) Log of minus the radion mass squared versus log of the warp factor, for the case  $\alpha = 1$ . (b) Outer (solid) lines: the radion wave function for a large value of the warp factor; inner (dashed) lines: wave function of the first KK excitation of the radion.

The radion wave function in the large warp factor regime is shown in figure 3b. If the 4-brane is moved even farther away (larger warp factor), the wave function retains the same form, since it stays flat in the region of large  $\tilde{r}$ . Although these plots were made for the case  $\alpha = 1$ , we find that the wave function looks essentially the same for all values of  $\alpha$ . We see from its functional form that near the 3-brane  $A_1 \cong -B_1$ , and since  $C_1$  is constrained to be  $3A_1 + B_1$ , therefore  $C_1 \cong -2B_1$  in this region. For larger values of  $\tilde{r}$  we have  $A_1 = 1/3$  and  $B_1 = -2/3$ , so  $C_1$  is extremely small throughout most of the bulk. Nevertheless its integral is nonvanishing, so the radial size of the extra dimension, which is given by  $\int_0^{\tilde{R}} d\tilde{r} C(\tilde{r})$ , changes in response to the instability, and it does so in the same sense as the size of the compact dimension, because of our choice of signs in the definitions of  $B_1$  and  $C_1$ . That is, the instability is a simultaneous growth or shrinking of the radius together with the circumference. Either direction is a possibility, since the static solution is analogous to sitting on the top of a hill: the ball can roll down in either direction. The situation is illustrated in figure 4. Also shown there is the fact that the relative size of the brane directions,  $x^\mu$ , grow or shrink in the opposite sense relative to the extra dimensions. In the case where the extra dimensions grow, the endpoint must be the AdS soliton solution with no 4-brane, since this has been demonstrated to be the minimum energy solution which is stable [23]. In the case where they shrink, the 2-D surface presumably degenerates into a point.

In comparing the radion in this model to that of the 5-D Randall-Sundrum model, we can notice several similarities and differences. Similarly to the 5-D model [28, 29], in 6-D the radion is an admixture of the radial and brane metric components, such that oscillations of the radial size are accompanied by fluctuations in the scale factor of the 4-D universe which are  $180^\circ$  out of phase. But in 5-D, the radion was exactly massless in the absence of stabilization, whereas in 6-D it is a tunable parameter. Another difference is that, whereas in 5-D the radion has no tower of KK excitations, in 6-D it does. The mass gap is of order  $1/L$ , *i.e.*, the TeV scale. The first few eigenvalues, for large values of the warp factor, are given in table 3. One notices that these masses are systematically smaller than those of the graviton and vector modes. Thus the

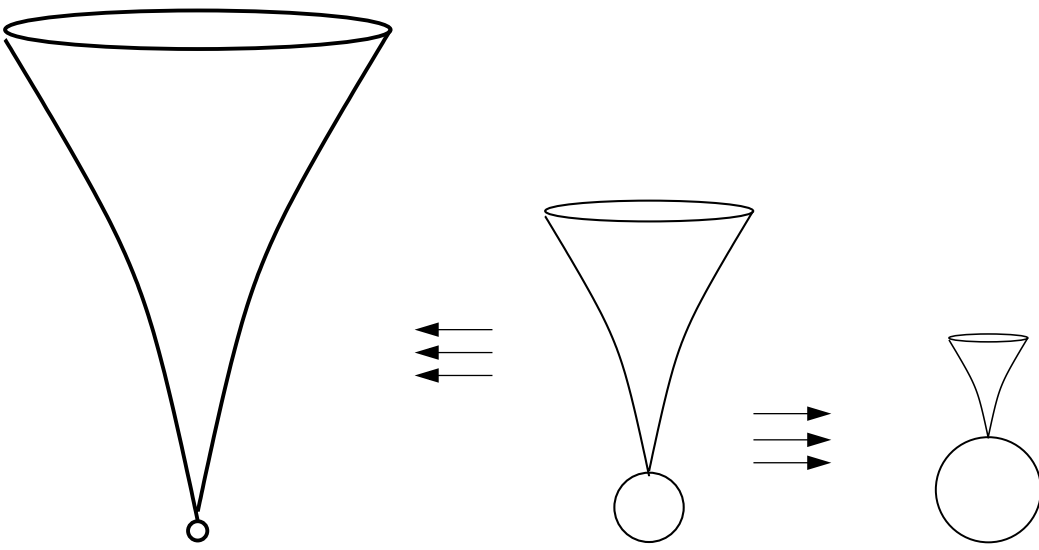


Figure 4. Illustration of the instability. A given static solution, shown in the center, is unstable toward growth (left) or shrinkage (right) of the extra dimensions (funnel), accompanied by the opposite behavior of the directions  $x^\mu$  within the 3-brane (shown as a sphere).

Mode Number	$m_r^2 L^2$
1	1.0188
2	6.1512
3	12.748
4	21.311
5	44.437
6	75.588

Table 3. Mass squared of the radion KK modes, in units of the AdS curvature radius, in the limit of large warp factor.

radion excitations would be the first signs of new physics from this model (once we have made it viable by stabilizing the radion) to appear in accelerator experiments. The wave functions of the second and third excited states are shown in figure 5, while that of the first excited state appears in figure 3b. Similarly to the spin 2 excitations, the modes above the ground state have wave functions which are exponentially strongly peaked on the TeV brane. The wave function of the radion ground state, on the other hand is relatively flat throughout the bulk. This plays an important role in its couplings, as we will discuss in the next section.

We can understand the preceding results for the radion ground state mass and wave function analytically, and generalize them to arbitrary values of the 4-brane stress-energy parameter  $\alpha$ . Toward this end, we first convert the coupled first order equations into a single second order equation. The form of the equations suggests that a natural dependent variable to consider is the linear combination  $H = 3A_1 + B_1$ . The variables  $A_1$ ,  $B_1$  and  $C_1$  can be expressed in terms of  $H$  using eqs. (43) and (44), where for later convenience we continue to show the effect of a bulk scalar field, even though we set it to zero for the present:

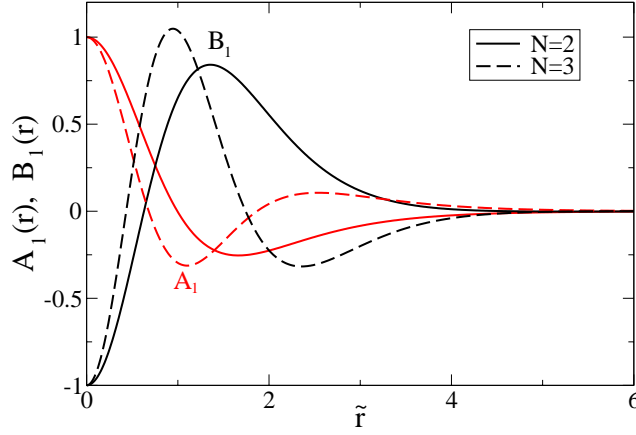


Figure 5. Solid lines: wave function of second excited state of the radion; dashed lines: the third excited state.

$$\begin{aligned}
A_1 &= \frac{1}{2B'_0} \left[ -H' + (A'_0 + B'_0) H - 2\kappa^2 \phi'_0 \phi_1 \right] \\
B_1 &= \frac{3}{2B'_0} \left[ H' - \left( A'_0 + \frac{1}{3} B'_0 \right) H + 2\kappa^2 \phi'_0 \phi_1 \right] \\
C_1 &= \frac{1}{2B'_0} \left[ H' + (B'_0 - A'_0) H - 2\kappa^2 \phi'_0 \phi_1 \right].
\end{aligned} \tag{55}$$

Substituting these into the remaining field equation (45), we obtain

$$\begin{aligned}
H'' + \frac{6A_0'^2 - 3B_0'^2 - 6A_0'B'_0 - 2\kappa^2 \phi_0'^2}{2B'_0} H' + \left( \frac{(B'_0 - A'_0)(3A_0'^2 + 2B_0'A'_0 - \kappa^2 \phi_0'^2)}{B'_0} + \frac{m_r^2}{a_0} \right) H \\
- 4\kappa^2 \left( \frac{\partial V}{\partial \phi} - \kappa^2 \frac{\phi'_0}{B'_0} V \right) \phi_1 = 0
\end{aligned} \tag{56}$$

and the boundary conditions at  $\tilde{r} = \tilde{R}$  and  $\tilde{r} = 0$ , respectively are

$$\begin{aligned}
\left( A'_0 + \frac{1-\alpha}{4} B'_0 + \frac{2\kappa^2 \phi_0'^2}{3(B'_0 - A'_0)} \right) H' &= \\
\left( \left( A'_0 + \frac{1}{3} B'_0 \right) \left( A'_0 + \frac{1-\alpha}{4} B'_0 \right) - \frac{2}{3} \kappa^2 \phi_0'^2 + \frac{2B'_0 m_r^2}{3(B'_0 - A'_0) a_0} \right) H, &\quad \tilde{r} = \tilde{R} \\
H' = A'_0 H = 0, &\quad \tilde{r} = 0
\end{aligned} \tag{57}$$

The radion mass squared comes into the boundary condition because, in the process of eliminating  $A_1$ ,  $A'_1$ ,  $B_1$  and  $B'_1$  in favor of  $H'$  and  $H$ , it is necessary to use the bulk equation (45), evaluated at the 4-brane.

Now it happens that the important features of the radion ground state can be deduced from approximating the above equations by the form which they take in the asymptotic region of  $\tilde{r} \sim \tilde{R}$  when  $\tilde{R}$  is large. In this region, we have

$$A'_0 \cong B'_0 \cong -\frac{4}{5}k; \quad B'_0 - A'_0 \cong -8ke^{-2k\tilde{r}} \equiv \delta A' \tag{58}$$

and the equations simplify considerably:

$$H'' - \frac{3}{2}A'H' + \left(5A'\delta A' + \frac{m_r^2}{a_0}\right)H = 0 \quad \text{in the bulk;} \quad (59)$$

$$\left(\frac{\kappa^2\phi_0'^2}{A'} + \frac{3}{8}(5-\alpha)\delta A'\right)H' = \left(\frac{m_r^2}{a_0} + \left(\frac{5-\alpha}{2}A' - \frac{\kappa^2\phi_0'^2}{A'}\right)\delta A'\right)H \quad \text{at } \tilde{r} = \tilde{R}, \quad (60)$$

where now we take  $A'$  to have the constant value  $-4k/5$ . The terms  $\delta A'$  and  $m_r^2/a_0$  are both of order  $e^{-2k\tilde{r}}$  in the large  $\tilde{r}$  region (as we will verify self-consistently), so that in the bulk equation (59) they can be ignored compared to the other terms. The solution in the bulk has the form

$$H(\tilde{r}) \cong c_1 + c_2 e^{-(6k/5)\tilde{r}} + \delta H, \quad (61)$$

where  $\delta H$  represents the small effect of the parenthetical terms we have ignored. The latter give rise to a negligible effect on the bulk solution,  $\delta H \ll H$ . However, the small terms proportional to  $\delta A'$  and  $m_r^2/a_0$  *cannot* be neglected when applying the b.c. at the 4-brane. In fact, this equation, (60), can be used to solve for the radion mass, which in the absence of the scalar field gives

$$\begin{aligned} m_r^2 &= \frac{5-\alpha}{4}a_0\delta A' \left(\frac{3}{2}\frac{H'}{H} - 2A'\right) \Big|_{\tilde{r}=\tilde{R}} \cong (\alpha-5)\frac{20}{2^{4/5}}L^{-2}e^{-(6k/5)\tilde{R}} \\ &= (\alpha-5)\frac{5}{L^2}a_0^{-3/2}(R) + O(a_0(\tilde{R})\delta A'^2) \\ &= \frac{\alpha-5}{2}\frac{\Lambda}{a_0^{3/2}(R)} + O\left(\frac{\Lambda}{a_0^4(R)}\right) \end{aligned} \quad (62)$$

The final expression assumes that  $a_0$  is normalized to unity at the TeV brane, and uses the relation (3) between  $\Lambda$  and  $L$ . (The intermediate factor of  $2^{4/5}$  comes from  $a_0(\tilde{R}) = \cosh^{4/5}(k\tilde{R})$ .) Interestingly, the value for  $m_r^2$  which we so obtain is completely insensitive to the details of  $\frac{H'}{H} \sim e^{-(6k/5)\tilde{R}}$ , much less  $\delta H$ , since all of these are much smaller than  $A'$ . We are therefore able to give a very accurate analytic estimate for the radion mass squared, when the warp factor is large. The small magnitude of  $m_r^2$  is seen to be a direct consequence of the value of  $B'_0 - A'_0$  in the static solution. This expression agrees with our previous numerical results for  $\alpha = 1$  (and we have also checked it numerically for other values of  $\alpha$ ). In the limit that the 4-brane goes to infinity, so that the full AdS soliton is recovered, the radion becomes massless, but is not normalizable. Thus it does not contradict the fact that the uncut AdS soliton is a stable solution.

Interestingly, the radion mass vanishes almost exactly in the case where the anisotropy of the 4-brane stress tensor is provided by the Casimir energy and pressure of fields living on the compact extra dimension. In this case the relevant energy density scales like  $L_\theta^{-5}$ , *i.e.*,  $\alpha = 5$ , as expected from dimensional analysis. This is the unique case where no dimensionful parameter is introduced in the anisotropic part of the 4-brane stress tensor, which is the part that also controls the position of the 4-brane, and hence how large the extra dimensions are. Curiously, the mass does not vanish *exactly* when  $\alpha = 5$  because of the  $O(\delta A'^2)$  correction, whose coefficient turns out to be  $(11+\alpha)/8$ . However, the natural size of this contribution to  $m_r$  is of order  $10^{-10}$  eV, which is far below experimental limits on scalar-tensor theories of gravity.

## 4 Radion stabilization and phenomenology

In this section we show how to increase the mass of the radion through using a bulk scalar field, and discuss the implications of the model for collider experiments, tests of the gravitational force, and cosmology.

### 4.1 Stabilization by a bulk scalar field

We have found that the radion can be massive, massless, or tachyonic, depending on the value  $\alpha$  which controls the dependence of the 4-brane stress-energy on its circumference. In the latter two cases ( $\alpha \leq 5$ ), it is certainly necessary to increase the radion mass squared so that we have a stable universe, with Einstein gravity rather than scalar-tensor gravity at low energies. In the 5-D RS1 model, this was achieved by Goldberger and Wise [18] by adding a bulk scalar field, whose VEV's at the two branes were constrained by potentials on the branes to take certain values. The bulk scalar then acts like a spring between the branes, whose gradient energy becomes repulsive if the branes get too close, and whose potential energy (from  $m^2\phi^2$ ) causes attraction if the branes separate too much. We expect that the same mechanism should work in 6-D.

Scalar fields in AdS have solutions which are exponentially growing or decaying toward the ultraviolet cutoff brane. Ref. [14] studied these solutions and found the approximate behavior

$$\phi(\tilde{r}) = \phi_+ e^{\sigma_+ \tilde{r}} + \phi_- e^{\sigma_- \tilde{r}} \quad (63)$$

where  $\sigma_{\pm} = -k \pm \sqrt{k^2 + m^2}$ . Near the 3-brane, where the space does not look like AdS, the behavior is different;  $\phi(\tilde{r}) \cong \phi_0(1 + m^2\tilde{r}^2/4)$ , but this will not be very important for understanding the effect of the scalar since most of the volume of the extra dimensions is near the 4-brane. For generic boundary conditions, the growing solutions dominate, and it is a good approximation to neglect the decaying ones. The main point is that the most natural configurations are ones where  $\phi(0) < \phi(\tilde{R})$ .

Before doing any analytic estimates, we solved the entire system of Einstein equations numerically, to find the effect of the scalar field on the radion mass. There are three kinds of corrections to consider. First, the scalar field induces a small back-reaction on the static solutions,  $A_0$ ,  $B_0$ , determined by the zeroth order truncation of the Einstein equations (38-42). This effect has been analytically computed in [14]. Second, the background scalar configuration couples to the fluctuations of the metric. This arises solely through the term  $\kappa^2\phi_0'^2 C_1$  of the perturbed ( $rr$ ) component of the Einstein equations, (45). We will see that this is the really important effect for stabilizing the radion. The third kind of correction is from fluctuations of the scalar field, which can mix with the radion. These are governed by the perturbed scalar field equation (46).

$$\phi_1'' - \frac{1}{2}(4A_0' + B_0')\phi_1' - \phi_0'H' - \frac{1}{2B_0'}\frac{\partial V}{\partial\phi}\left((B_0' - A_0')H + H' - 2\kappa^2\phi_0'\phi_1\right) - \frac{\partial^2 V}{\partial\phi^2}\phi_1 + \frac{m_r^2}{a_0}\phi_1 = 0 \quad (64)$$

where  $V = \frac{1}{2}m^2\phi^2$  is the bulk potential.

Our numerical results demonstrating the stabilization of the radion are shown in figure 6. We considered scalar field configurations with  $\phi = 0$  at the 3-brane and varied the value of  $\phi$  at the 4-brane, showing that  $m_r^2$  (in the tachyonic case  $\alpha = 1$ ) becomes positive for sufficiently large values of  $\phi(\tilde{R})$ . (Treating  $\phi(\tilde{R})$  as an adjustable parameter can be justified by imagining that we have stiff potentials for  $\phi$  on the branes, fixing their boundary values to whatever we desire.) We checked that these results are quite insensitive to whether the fluctuation of the scalar are included. The mixing between the radion and  $\phi_1$  was found to be negligible.

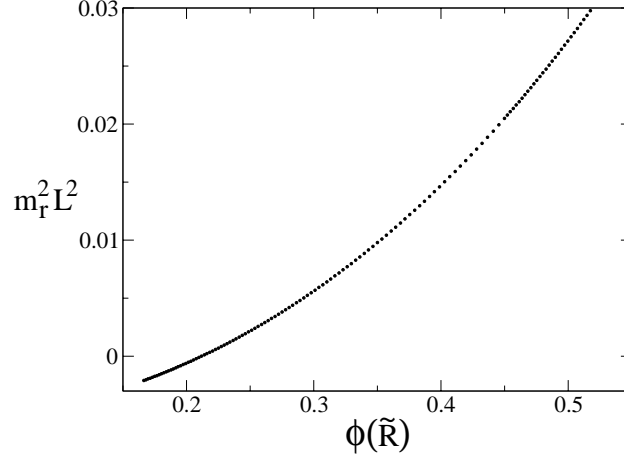


Figure 6. Dependence of  $m_r^2$  on the value of  $\phi$  at the 4-brane.

The behavior shown here can be easily understood by generalizing our previous derivation of the radion mass to include the effect of the scalar. The equation of motion and boundary condition for  $H$ , in the large- $\tilde{r}$  region, were given in (59-60). We can take  $m^2$  of the scalar to be small, so that its effect can be neglected in the bulk equation and the approximate solution (61) is still valid. Now when we solve the b.c. for the radion mass, we obtain the previous expression plus a new term,

$$\frac{m_r^2}{a_0} = \frac{5 - \alpha}{4} \delta A' \left( \frac{3}{2} \frac{H'}{H} - 2A' \right) + \frac{H'}{H} \frac{\kappa^2 \phi_0'^2}{A'} \quad (65)$$

whose origin can be traced to the extra term in the  $(rr)$  Einstein equation. The first term is also changed by the presence of the scalar field, because of its back-reaction on the static metric. However, using the results of ref. [14] who computed this back-reaction, we find that  $\delta A'$  is still of order  $e^{-2k\tilde{R}}$ . Therefore, since  $\frac{H'}{H}$  is of order  $e^{3k\tilde{r}/2}$ , the new term on the r.h.s. of (65) is the dominant one. The fact that  $\frac{H'}{H}$  has the correct sign (negative) to insure that the radion mass squared is positive is not obvious, but by numerically solving for  $H(r)$  we have verified that indeed  $\frac{H'}{H}(\tilde{R}) < 0$ , as we show in figure 7. We thus find that for large enough values of  $\phi(\tilde{R})$ , the radion mass is

$$m_r \sim \frac{m^2 \phi(\tilde{R})}{k M_6^2} e^{-k\tilde{R}/5} \sim \text{MeV} \quad (66)$$

independently of the details of the 4-brane stress-energy.

It is remarkable that the stabilized mass is not of order the TeV scale, as was the case in 5-D RS1 [18, 29, 30]. The mass squared is suppressed by the fractional power of the warp factor left in the product  $a_0 \frac{H'}{H} \sim a_0^{-1/2}$ . Recalling that the Planck scale hierarchy was set by  $a_0^{3/2} \cong 10^{32}$ , we see that the stabilized  $m_r$  is suppressed by the factor  $10^{16/3}$ , giving  $m_r \sim 10$  MeV. This is precisely the same factor by which a TeV mass particle, transported from  $\tilde{r} = 0$  to  $\tilde{r} = \tilde{R}$ , falls short of the Planck scale, as we noted in section 2.2. Thus the smallness of the radion mass seems to be associated with the additional dilution of the strength of gravity that comes from the large extra dimension.

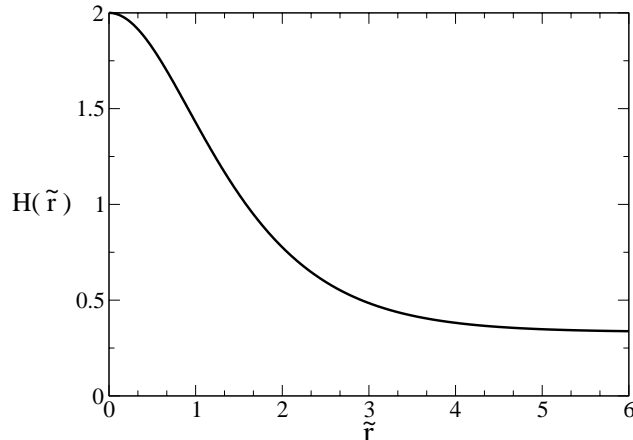


Figure 7. The radion wave function  $H(r)$  for the ground state, showing that  $H'/H < 0$ .

In ref. [14], one advantage of having a bulk scalar field was already pointed out: because of the back-reaction of the scalar on the metric, the jump conditions at the 4-brane can generically be satisfied for large values of  $\tilde{R}$ , as desired for achieving a large hierarchy, without much sensitivity to the model of stress energy on the 4-brane. In particular, choices like  $\alpha = 0$  (pure tension) or  $\alpha = 1$  (smeared 3-brane), which by themselves could not have yielded a satisfactory value of  $\tilde{R}$ , become viable in the presence of the bulk scalar. The only requirement for getting sufficiently large  $\tilde{R}$  is that the scalar mass should be somewhat light since

$$\tilde{R} \sim \frac{k}{m^2}. \quad (67)$$

Hence one needs  $k/m \sim 8$  to get the desired hierarchy. A similar relation occurs in the 5-D realization of Goldberger and Wise [18]. This requirement is quite compatible with the parameters we need for generating the radion mass, as in retrospect one would have expected.

## 4.2 Couplings of the radion and its excitations

In the 5-D RS1 model, the stabilized radion has a TeV scale mass and TeV suppressed couplings to standard model matter [31]. Were the couplings of our MeV-scale radion so large, it would easily be observable in low-energy experiments and possibly affect the cooling of supernovae. Here we show that the couplings are actually Planck scale suppressed.

Computing the 4-D effective Lagrangian for the gravitational fluctuations at quadratic order, we obtain

$$\mathcal{L}_0 = \frac{1}{3\kappa^2} \int d\theta dr a_0(r) \sqrt{b_0(r)} \left[ \dot{H}^2(r, t) + 2\dot{B}_1^2(r, t) \right] + A_1(0, t) T^\mu_\mu \quad (68)$$

where  $T^\mu_\mu$  is the trace of the 3-brane stress-energy tensor, representing the standard model,  $H(r, t) = H(r)\varphi_0(t)$ ,  $B_1(r, t) = B_1(r)\varphi_0(t)$ , *etc.* Here  $\varphi_0(t)$  represents the 4-D ground state radion field, and  $H(r)$ ,  $B_1(r)$ ,  $A_1(r)$  are the corresponding wave functions found in the previous section. Since they are nearly constant throughout the bulk, we can take them out of the integral and perform it to obtain

$$\mathcal{L}_0 \sim M_p^2 \dot{\varphi}_0^2 + \varphi_0 T^\mu_\mu \quad (69)$$

This shows that the canonically normalized radion field ground state has Planck-suppressed couplings to TeV-brane matter. This differs from the behavior of the radion in the 5-D RS1 model. There, the wave function of the radion is exponentially peaked at the Planck brane, which overcomes the exponential warp factor in the measure to give  $\mathcal{L}_0 \sim (\text{TeV})^2 \dot{\varphi}_0^2 + \varphi_0 T^\mu_\mu$  instead. The flatness of the radion wave function in the present case accounts for its weak couplings to the TeV brane.

The KK excitations of the radion *are* exponentially peaked on the TeV brane, on the other hand. We can understand this from the asymptotic form of the bulk equation of motion (59); since the mass is no longer negligible, the solutions behave like  $H(r) \cong c_2 e^{-6k\tilde{r}/5}$ , with the constant piece  $c_1$  equal to zero. The integrand of (68) behaves like  $e^{-12k\tilde{r}/5 + 12k\tilde{r}/5} = O(1)$ , so we obtain

$$\mathcal{L}_n \sim \frac{M_6^4}{k} \tilde{R} \dot{\varphi}_n^2 + \varphi_n T^\mu_\mu \quad \rightarrow \quad \dot{\varphi}_n^2 + M_6^{-2} \sqrt{\frac{k}{\tilde{R}}} \varphi_n T^\mu_\mu \quad (70)$$

Hence the coupling of the radion excited state is suppressed only by the small factor  $(M_6 \tilde{R})^{1/2} \sim \sqrt{60}$  relative to the TeV scale. The radial KK gravitons have similar couplings, but larger masses (compare Tables 1 and 3), so the radion excitations would be the first signal of new physics in collider experiments. Heavy radions could be copiously produced in the *s*-channel at the LHC, through gluon-gluon fusion events due to the QCD trace anomaly contribution to  $T^\mu_\mu$ .

### 4.3 Gravity and cosmology

Although perhaps less physically motivated, models of the 4-brane stress-energy with  $\alpha > 5$  predict that the radion mass squared will be positive even without a bulk scalar, and that its magnitude is in the  $10^{-3}$  eV regime. This is within the reach of Cavendish-type tests of submillimeter gravity [32], and will be even more accessible to upcoming versions of the experiment which will have improved sensitivity.

It may also be possible to achieve this situation without appealing to exotic forms of matter on the 4-brane. The radion mass will get radiative corrections from its couplings to matter. Since the radion couples to the trace of the stress energy tensor on either brane, the heaviest particles will contribute the most strongly. Considering matter which is on the TeV brane, we can estimate the size of the one-loop correction as

$$m_{r,1\text{-loop}}^2 \sim \frac{\text{TeV}^4}{M_p^2} \quad (71)$$



The numerator comes from the fact that the TeV scale is the cutoff on the 3-brane, and the heaviest particles will have masses of this order, whereas the denominator is due to the fact that the lowest mode of the radion has Planck-suppressed couplings. This argument could be upset if the 4-brane has massive particles which are much heavier than the TeV scale, since by the same argument these could apparently make the radion very heavy and presumably would destabilize the hierarchy which we have achieved. It may be necessary to assume that there are only massless particles on the 4-brane to avoid this.

On the other hand, if we allow heavy particles to exist on the 4-brane, their natural mass scale is  $\sqrt{a(R)}$  TeV  $\sim 10^{13}$  GeV. It is interesting that this is the right order of magnitude for generating the observed primordial density fluctuations from the simplest model of chaotic inflation. This is an advantage of the present model over the 5-D RS model, where a  $\sim 10^{13}$  GeV would look unnaturally light were it living on the Planck brane, and of course too heavy to exist on the TeV brane.

## 5 Summary

We have focused on the simplest and most direct generalization to six dimensions of the 5-D Randall-Sundrum two-brane model: the AdS soliton model, with the TeV 3-brane at the center of the azimuthally symmetric extra dimensions, and a 4-brane cutting the space off at some finite radius. The model has many features in common with its 5-D predecessor: the geometry is highly warped and very close to AdS in the region far from the 3-brane, the graviton zero-mode is localized on the hidden brane, while the radial KK excitations are localized on the TeV brane and have a TeV mass gap. In both models, the radion can easily be stabilized by the Goldberger-Wise mechanism, using a bulk scalar field.

However, there are also some quite distinctive differences. The hierarchy between the Planck and weak scales, while generated mostly (2/3) by warping, is also partly (1/3) due to the exponentially large size of the compact extra dimension [33], giving it some features in common with the large extra dimension proposal. The mass scale at the 4-brane is not the Planck scale, but it is suppressed by the size of the large compact dimension to the  $10^{13}$  GeV scale. There is a tower of relatively light ( $\sim$  TeV) KK gravitons corresponding to this large dimension. In the absence of stabilization by a scalar field, the 6-D model requires some mildly exotic form of stress energy on the hidden brane in order to have a finite volume. The 4-brane stress tensor generically depends on the size of the extra dimension as  $L_\theta^{-\alpha}$  with some model-dependent number  $\alpha$ .

Most of these features were already known; in the present work we computed the spectrum of metric perturbations, including the graviton and graviphoton modes, and we found the unexpected new result that the radion is not necessarily massless, but has a mass squared which depends linearly on  $\alpha$  and the negative bulk cosmological constant:  $m_r^2 \sim (5 - \alpha)\Lambda$  TeV/ $M_p$ . Only for the special case of Casimir energy on the 4-brane ( $\alpha = 5$ ) is it massless. For smaller values of  $\alpha$  it is tachyonic, and the spacetime is unstable. Its couplings to the TeV brane are Planck suppressed rather than TeV suppressed, due to the different behavior of its wave function relative to the 5-D case. Once stabilized by a bulk scalar field, the radion mass

is not TeV scale, as in 5-D, but rather at the MeV scale. This suppression is related to the presence of the large extra dimension which does not feature in 5-D.

The 6-D model has similar phenomenology to the 5-D model, since the Kaluza-Klein excitations of the radion behave much like the ground state of the stabilized 5-D radion. However, there is a new possibility that the radion is stabilized not by the Goldberger-Wise mechanism, but by some form of stress energy on the 4-brane which has  $\alpha > 5$ , or perhaps by radiative corrections from standard model particles on the TeV brane. In this case the radion mass is in the milli-eV range, which is just right for being accessible in experiments which test gravity below 1 millimeter.

The latter possibility would seem to require the absence of massive particles on the 4-brane, since radiative effects there should induce much larger corrections to the radion mass. In fact it might be necessary to forbid heavy particles on the 4-brane just to maintain the large hierarchy we set out to achieve. This is a question which deserves further study. But if it is consistent to have heavy fields on the 4-brane, then the fact that their mass is naturally of order  $10^{13}$  GeV is intriguing for inflation, since this is the right scale for getting density perturbations of order  $10^{-5}$  in chaotic inflation.

We have left for future work a study of 3-brane fluctuation modes, where the position of the 3-brane could oscillate with respect to the center of the extra dimensions. These modes, if they exist and are sufficiently light or strongly coupled, could be important for the phenomenology of this model, since they might induce a coupling between the graviphoton and standard model particles.

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